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Vu Trieu Minh*, Tamre Mart, Reza Moezzi, Mets Oliver, Jurise Martin, Polder Ahti, Teder Leo, and Juurma Mart

Performances of PID and Different Fuzzy Methods for Controlling a Ball on Beam

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Abstract: This paper develops and analyses the performances evaluation of different control strategies applied for a nonlinear motion of a ball on a beam system. Comparison results provide in-depth comprehension on the stable ability of different controllers for this real mechanical application. The three different controllers are a conventional PID method, a Mamdani-type fuzzy rule method and a Sugeno-type fuzzy rule method. In this study, the PID shows the fastest sinuous reference tracking while the Mamdani-type fuzzy method proves the highest stability performance for tracking square wave motions.

Keywords: PID Control; Fuzzy Logic Control; Mamdani rule; Sugeno rule; Motion Tracking

1 Introduction

Fuzzy logic control is one of the intelligent methods based on uncertain information and human responses. In complicated and nonlinear systems, the use of fuzzy logic can avoid the construction of precise mathematical models since fuzzy logic can provide very successful control performances based on uncertain and imprecise inputs. An example for the application of fuzzy logic for control of clutch slip and vibration can be referred in reference [1]. Even though there are always available mechanical models for the automotive clutch engagements but due to the too complex interactions between the clutch and the driver for all driving conditions (such as loads, roads, speeds, etc.), the use of fuzzy logic control is always the best choice applied for vehicle automated clutch engagements and for anti-lock braking systems.

This paper investigates the control performances of a conventional PID with different fuzzy methods and it gives an in-depth understanding of the performances/stability of fuzzy for a ball on beam system. The balance and the motion control of a ball on a beam are always the challenges for both conventional and intelligent control strategies. This system is widely illustrated because of its simplicity and tangibility to evaluate the control performances and the system stability. In this research, a PID controller and two fuzzy methods are also designed and tested.

Researches on the stability analysis of different fuzzy methods are still few since all fuzzy strategies do not use the mathematical models but heuristic fuzzy rules. Therefore, the stability of a fuzzy system cannot be determined by a Lyapunov mathematical function. This paper reviews recent researches using fuzzy control position of a ball on beam, one of the most popular models for teaching control engineering in universities.

There are few researches on the comparison of different fuzzy methods for this system. Reference [2] presents the control simulation of a ball on beam using fuzzy static and fuzzy dynamic methods. Simulations show that the static fuzzy can control the ball faster but the dynamic fuzzy provides lower overshoot error. Reference [3] introduces the design of a fuzzy logic control of ball on beam, and compares it with a PID. Simulation results show that the PID provides faster response but the fuzzy achieves lower overshoot error. Reference [4] developed the design of different PID controllers and compared them to the fuzzy logic control. Similarly, simulations show that the fuzzy provides better performance on overshoot but slower transient time.

Regarding the stable ability of a fuzzy system, some researchers have attempted to use dual-systems. The outer loop uses fuzzy, and the inner loop uses PID with poles replacement [5]. The stability is guaranteed but the use of PID for the inner loop to re-locate the poles from the outer loop by the fuzzy rules will deteriorate or eliminate

***Corresponding Author: Vu Trieu Minh:** Tallinn University of Technology, Tallinn, Estonia; Email: vutrieuminh@yahoo.com
Tamre Mart, Reza Moezzi, Mets Oliver, Jurise Martin, Polder Ahti, Teder Leo, Juurma Mart: Department of Mechatronics, Faculty of Mechanical Engineering, Tallinn University of Technology, Ehitajate tee 5, 19086, Tallinn, Estonia

all fuzzy rules. Reference [6] proposes the use of adaptive control to ensure the stability of Lyapunov function and a fuzzy controller. Similar to a dual-controller system, the inner loop is for an adaptive controller and the outer loop is for a fuzzy controller. Also, the stability of the whole system is guaranteed by the adaptive control law.

Reference [7] presents a combination of a genetic algorithm (GA) approach and fuzzy control, however the system is complicated and the performances are slow. Most of recent researches using dual-systems and/or sliding modes to ensure the Lyapunov function did not take into account the fact that even all control modes are stable but the switching sequences among those modes can destabilize the whole system [8]. It means that even if all controllers are stable, the switching among those stable controllers can lead to instability. Therefore, it is very important to find a common Lyapunov function for all separate controllers. This common Lyapunov will guarantee the stability for all switching sequences among those controllers. The system modelling is referred in [9] and [10].

This paper presents the design of three controllers: PID, Mamdani-type fuzzy, Sugeno-type fuzzy and tests their performances. Then, the paper suggests which controllers provide better performances. Simulation results of each control method is demonstrated and analysed. The contents of this paper are as follows: Section 2 presents the system modelling; Section 3 introduces the design of PID controller; Section 4 develops the Mamdani-type fuzzy control; Section 5 constructs the Sugeno-type fuzzy control; while Section 6 states the conclusions on this research.

2 Mathematical modelling

The motion of a ball on beam is an unstable and nonlinear motion and therefore inherits the most popular challenge in teaching control. This system is demonstrated in Figure 1 including a ball rolling along a beam. The beam is connected to a shaft of an electrical motor with a distance (d). Then, the beam can tilt in order to control the ball position. Position of the ball with distance (r) rolling on the beam length (L), the beam angle alpha (α) and the motor shaft angle theta (θ) can be measured and controlled.

It is assumed that the ball can roll on the beam without any slipping. By using a model from the Lagrangian method of energy balance in [9], the Lagrangian of a system (L) is the subtraction of the Kinetic (K) and the Potential Energy (U):

$$L = K - U \tag{1}$$

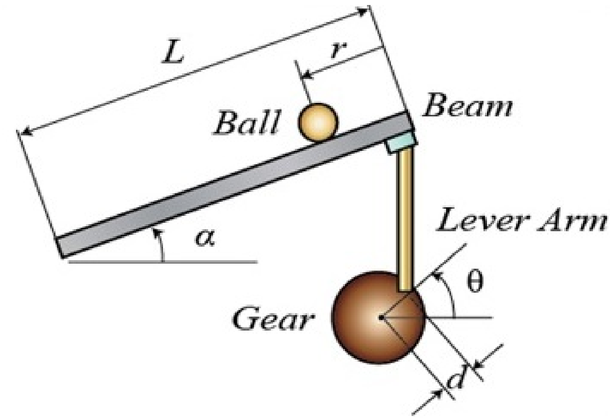

Figure 1: Ball and Beam Model.

The Kinetic Energy of this system is the sum of Kinetic Energy of the beam (K_1) and the Kinetic Energy of the ball (K_2):

$$K = K_1 + K_2 \tag{2}$$

The Kinetic Energy of the beam:

$$K_1 = \frac{1}{2} J\dot{\alpha}^2 \tag{3}$$

where J is the moment of inertia of the beam.

The Kinetic Energy of the ball:

$$K_2 = \frac{1}{2}\left(\frac{J_b}{R^2} + m\right)\dot{r}^2 + \frac{1}{2} mr^2\dot{\alpha}^2 \tag{4}$$

where J_b is the moment of inertia of the ball and R is the radius of the ball, m is the mass of the ball.

The Potential Energy of the system:

$$U = mgr\sin\alpha \tag{5}$$

where g is the gravity constant.

Substituting (2), (3), (4), and (5) into (1), the Langrangian of this system is:

$$L = \frac{1}{2}\left(\frac{J_b}{R^2} + m\right)\dot{r}^2 + \frac{1}{2}\left(mr^2 + J\right)\dot{\alpha}^2 - mgr\sin\alpha \tag{6}$$

Apply the first Lagrange rule, the motion equation of the ball on the beam is:

$$\left(\frac{J_b}{R^2} + m\right)\ddot{r} + mg\sin\alpha - mr\dot{\alpha}^2 = 0 \tag{7}$$

The linearization of the system in (7) can be achieved at the angular velocity, $\dot{\alpha} \approx 0$, then:

$$\ddot{r} = \frac{-mg\sin\alpha}{\left(\frac{J_b}{R^2} + m\right)} \tag{8}$$

The beam angle alpha (α) and the motor shaft angle theta (θ) are related by the mechanic connection:

$$\alpha L = \theta d \tag{9}$$

The control of the ball position in (8) by the beam angle alpha (α) can be connected by the control of the motor angle theta (θ) in (9). Equations in (8) and (9) are used to develop the different controllers by using Matlab and Matlab Simulink version 2016 for the all next sections.

3 Design of PID controller

The motor angle theta (θ) determines the ball acceleration ($\ddot{r}$) by the Lagrangian equation (8), then going through an integrator → the ball velocity ($\dot{r}$), and going through another integrator → the ball position output (r) as shown in Figure 2.

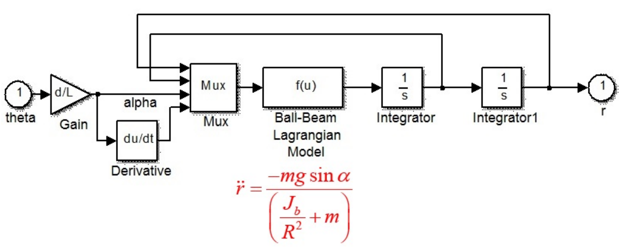

Figure 2: System Dynamics Modelling.

From this system model, two PID controllers are designed: one PID controller for the motor shaft angle theta (θ) in the inner loop, and another PID controller for the outer loop as shown in Figure 3. The first PID controller will support the out loop feedback while the second PID controller depends on the first PID controller. The system becomes more stable since the input signal for the second PID controller in the outer loop is provided by the first PID controller.

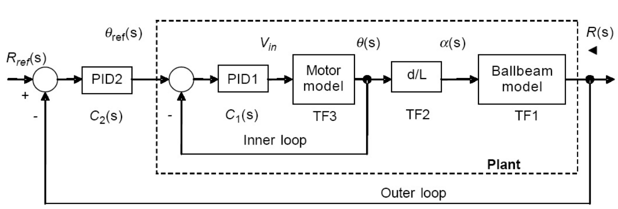

Figure 3: Design of a PID controller.

The following parameters and data are used for the whole simulations in this paper: Mass of the ball (m) of 0.11 kg; Radius of the ball (R) of 0.015 m; Lever arm offset (d) of 0.03 m; Gravitational acceleration (g) 9.8 m/s^2; Length of the beam (L) of 1.0 m; Beam moment of inertia (J_L) of 9.99e-6 kg·m^2; Ball moment of inertia of $J_b = 2mR^2/5$. Construction of a PID controller in Matlab Simulink is shown in Figure 4.

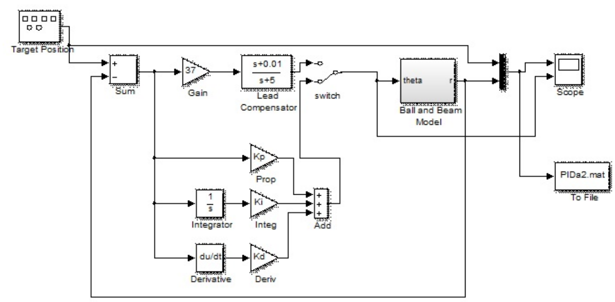

Figure 4: Matlab Simulink PID controller.

The PID system ($Kp = 5$; $Kd = 15$; and $Ki = 0.1$) is tested for the ball position (r) tracking a sinuous wave frequency from low to high. The tracking performances of the PID controller become worse at higher frequency. Figure 5 shows the PID tracking performance for a sinuous wave at amplitude of 1 and frequency of 0.8 rad/sec. The overshoot has increased to more than 15%.

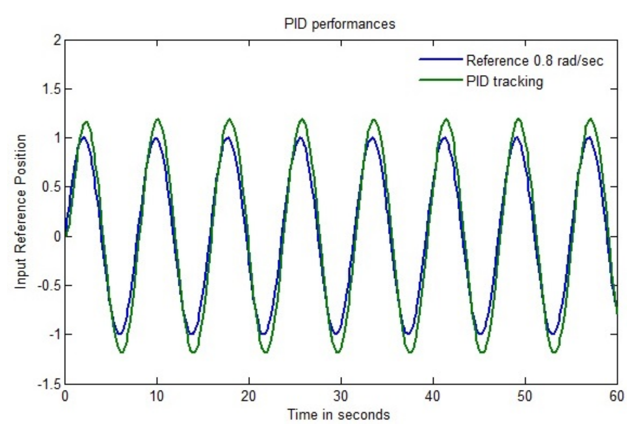

Figure 5: PID tracking performance.

The PID is destabilized after 40 secs for tracking a sinuous wave frequency of 0.81 rad/sec as shown in Figure 6. The PID controller cannot perform tracking of any square wave due to the singularity in its integrators to converse acceleration and velocity to its positions.

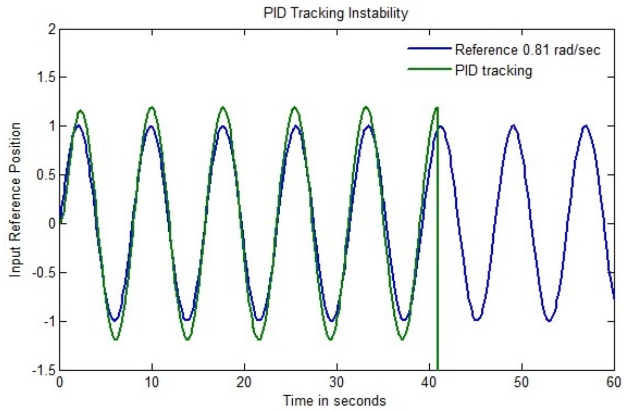

Figure 6: PID controller instability.

Fuzzy controllers will be built in the next parts and compared to the performances of this PID controller.

4 Mamdani-type fuzzy controller

Two fuzzy controllers were developed and compared. The inputs for the fuzzy control is the position error and the velocity of the error generated from the tracking performances. The control output is the angle of the beam angle alpha (α) and/or the motor shaft angle theta (θ) in (8) and (9). A Mamdani fuzzy logic controller in Matlab Simulink was designed as shown in Figure 7.

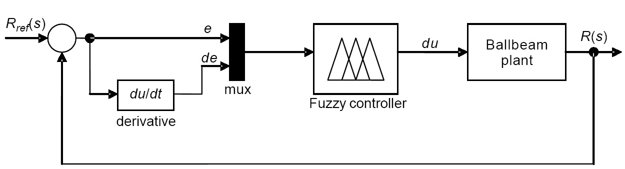

Figure 7: Fuzzy logic controller.

Mamdani is the most popular among fuzzy methods since it is intuitive, suitable for the human behaviours, and easy to develop. This method is based on the simple logic rules. For example: If x is A or/and y is B, then z is C. As mentioned earliar that the fuzzy control does not need any mathematical model. The inputs will be fuzzificated as fuzzy sets. Fuzzy rules are developed, based on the fuzzy operator (OR or AND). Then, aggregation of the rule outputs is proceeded, and finally, defuzzification is taken place. The diagram of Mamdani is shown in Figure 8.

The membership function of the inputs and output of this Mamdani fuzzy is described in Table 1.

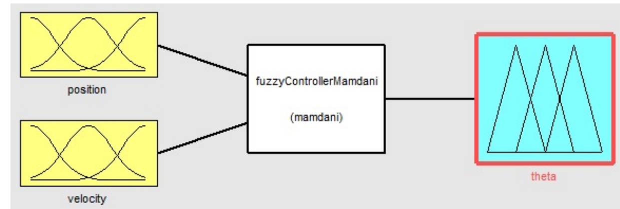

Figure 8: Fuzzy Logic Mamdani.

Performance of Mamdani fuzzy and the above PID for tracking a sinuous wave frequency of 0.2 rad/sec is illustrated in Figure 9. It shows that the fuzzy Mamdani responses lower and higher overshoot than PID at the starting time. But the overshoot error of the fuzzy controller will become lower than PID after 15 seconds.

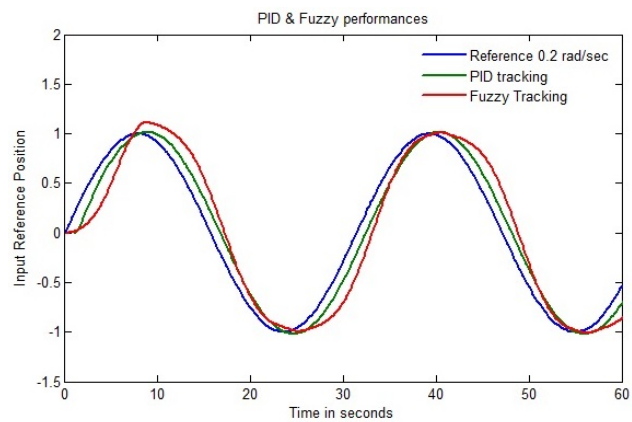

Figure 9: PID and Fuzzy Mamdani performances.

As indicated in Figure 6 that the PID tracking performance will be destabilized at frequency of 0.81 rad/sec after 40 seconds while the Mamdani fuzzy control is still maintained well stability. However, the tracking error becomes larger as the Mamdani responses slower as shown in Figure 10.

Next part, another fuzzy method namely Sugeno is designed and compared to this Mamdani fuzzy.

5 Sugeno-type fuzzy controller

Sugeno fuzzy method is more compact and more computationally effective than Mamdani because Sugeno applies the use of adaptive control for constructing its fuzzy rules. Also, this method is based on the linearization of fuzzy memberships. In this part, a Sugeno fuzzy controller is de-

Table 1: Mamdani fuzzy rule values.

Mamdani codes	Position (P)	Velocity (dP)	Output Theta
NB: negative big	[−1.2 −1 −0.45 −0.2]	[−2.9 −1.9 −0.9 −0.4]	[−8 −7.5 −2.5 −1.5]
NM: negative medium	[−0.45 −0.2 −0.05]	[−0.9 −0.4 −0.2]	[−2.2 −1.2 −0.2]
NS: negative small	[−0.2 −0.05 0]	[−0.4 −0.1 0]	[−0.7 −0.2 0]
ZR: Zero	[−0.025 0 0.025]	[−0.05 0 0.05]	[−0.25 0 0.25]
PS: positive small	[0 0.05 0.2]	[0 0.1 0.4]	[0 0.2 0.7]
PM: positive medium	[0.05 0.2 0.45]	[0.2 0.4 0.9]	[0.25 1.2 2.2]
PB: positive big	[0.2 0.45 0.95 1.45]	[0.4 0.9 1.9 2.9]	[1.5 2.5 7.75 8]

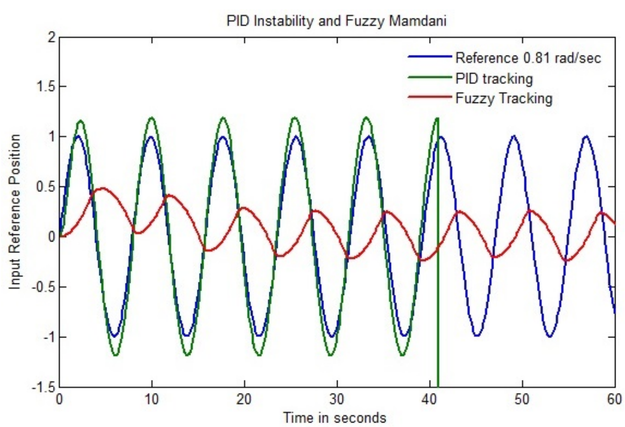

Figure 10: PID Instability and Fuzzy Mamdani.

signed as shown in Figure 11, to compare with the Mamdani fuzzy.

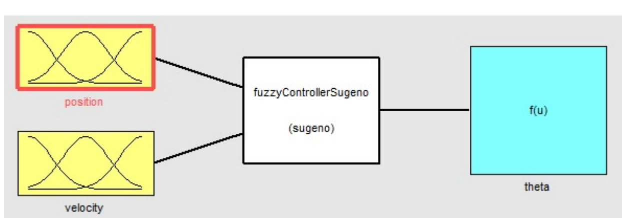

Figure 11: Fuzzy Logic Sugeno.

In Sugeno, the fuzzy rules are normally defined as: If x is A or/and y is B, then $z = ax + by + c$, as a linear equation. For a Sugeno of zero order, the output z will be a constant as $a = b = 0$. The Sugeno provides better applications for mathematical analysis as can be seen here. In this Sugeno design, the two inputs are the ball positon (P) and the ball velocity (dP), while the only output is Theta $= a \cdot P + b \cdot dP + c$, where a, b, c are the coefficients calculated and shown in Table 2.

Since the PID cannot track the square wave, the two fuzzy methods are now tested for only square waves to indicate the superiority of fuzzy over PID. Figure 12 shows the comparison of Mamdani and Sugeno tracking a square wave amplitude of 0.5 and frequency of 0.1 rad/sec. Although both methods perform the tracking very well, Sugeno generates a little bit higher overshoot and slower transient time.

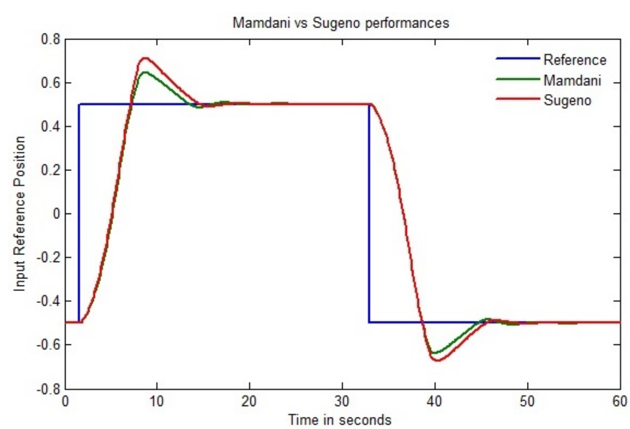

Figure 12: Fuzzy Mamdani vs Sugeno.

Then, the amplitude of the square wave is gradually increasing to test which fuzzy will be destabilized first. Figure 13 shows that at the amplitude of 1.03, Sugeno is destabilized and jumps out of the tracking reference after 40 seconds, while Mamdani still performs very well it tracking performance. It is also noted that Sugeno responds faster in transient time and higher overshoot, while Mamdani looks slower, lower overshoot and more stable.

Finally, the amplitude of the reference wave is increased to test the limit that the Mamdani is destabilized. Figure 14 shows at the square wave amplitude of 3.1, the Mamdani fuzzy becomes destabilization and jumps out after 52 seconds, while the Sugeno had jumped out already from the tracking performance after only 10 seconds.

In all simulations, Mamdani always shows its best performances and achieves the highest level of stability over

Table 2: Sugeno fuzzy rule values.

Sugeno codes	Position (P)	Velocity (dP)	Theta = $a \cdot P + b \cdot dP + c$
NB: negative big	[−1.2 −1 −0.45 −0.2]	[−2.9 −1.9 −0.9 −0.4]	[0.1 0. −3.5]
NM: negative medium	[−0.45 −0.2 −0.05]	[−0.9 −0.4 −0.2]	[0 0. −1.2]
NS: negative small	[−0.2 −0.05 0]	[−0.4 −0.1 0]	[0.1 0. −0.3]
ZR: Zero	[−0.025 0 0.025]	[−0.05 0 0.05]	[0.1 0. 0.]
PS: positive small	[0 0.05 0.2]	[0 0.1 0.4]	[0. 0. 0.3]
PM: positive medium	[0.05 0.2 0.45]	[0.2 0.4 0.9]	[0. 0. 1.2]
PB: positive big	[0.2 0.45 0.95 1.45]	[0.4 0.9 1.9 2.9]	[0. 0. 3.2]

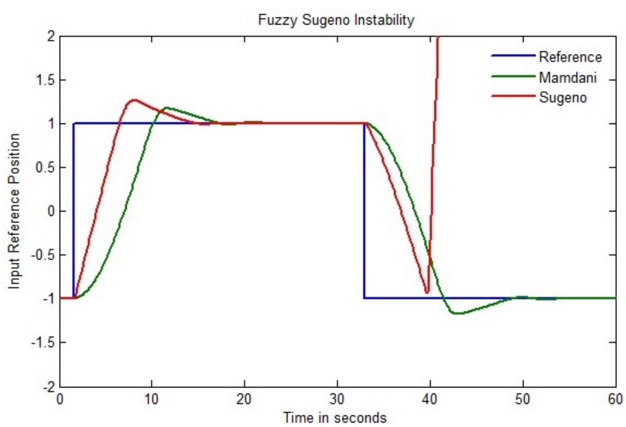

Figure 13: Fuzzy Sugeno Instability.

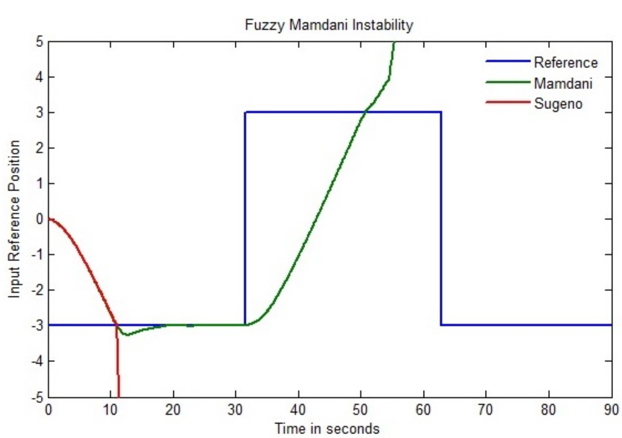

Figure 14: Fuzzy Mamdani Instability.

Sugeno. Even though, Mamdani seems having a little bit slower response in transient time.

6 Conclusions

This study shows the superiority of fuzzy methods over PID for tracking square waves due to the singularity in the integrators at PID. Therefore, initial conditions for integrators in PID must be regulated and changed to avoid this singularity. For the two fuzzy methods, Mamdani has proved to be the most popular used among other fuzzy methods because it is more suitable for human behaviours and easier to be developed. Sugeno is also a good fuzzy selection since it can work well with linear equations in its rules and also applies adaptive techniques.

Conflict of Interests: The authors declare that there is no conflict of interest regarding the publication of this research article.

References

[1] Minh, V.T., Pumwa, J., Fuzzy logic and slip controller of clutch and vibration for hybrid vehicle, International Journal of Control, Automation and Systems, 2013, 11(3), 526–532

[2] Muawia A, Nordin B, Rosdiazli B, Simulation of a ball on a beam model using a fuzzy dynamic and a fuzzy static sliding mode controller, Research Journal of Applied Sciences, 2014, 8(2), 288–295

[3] Amjad M, Kashif M, Abdullah S, Fuzzy logic control of ball and beam system, in Proc. International Conference on Education Technology and Computer (ICETC), China, 2010, 3, 489–493

[4] Herman W, Mohd F, A study of different controller strategies for a ball and beam system, Jurnal Teknologi, 2009, 50(D), 93–108

[5] Houshyar A, Arash M, Maysam O., Stabilization ball and beam by fuzzy logic control strategy, in Proc. International Conference on Machine Vision (ICMV), Singapore, 2012, 8349, 1–7

[6] Bhushan B, Valluru K, Singh M, Takagi-Sugeno Fuzzy system based stable direct adaptive control of nonlinear systems, International Journal of Computer Applications, 2013, 68(15), 30–36

[7] Tzeng S, GA approach for designing fuzzy control with nonlinear ball and beam system, in Proc. International Conference on Electrical Engineering (ICEE), Pakistan, 2008, 5, 1–8

[8] Minh, V.T, Stability for switched dynamic hybrid systems, Mathematical and Computer Modelling, 2013, 57(1–2), 78–83

[9] Carlos G., Gerson B., Modelling the ball and beam system from Newtonian mechanics and from Lagrange methods, in Proc.Latin American and Caribbean Conference for Engineering and Technology (LACCEI), Ecuador, 2014, 1, 1–9

[10] Minh VT, Afzulpurkar N, Wan Muhamad, Fault detection model-based controller for process systems, Asian Journal of Control, 2011, 13(3), 382–397