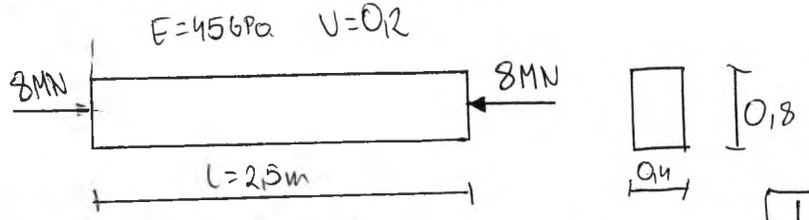
Obliczyć całkowitą energię potencjalną odkształcenia objętościowego i postaciowego



$\sigma_x = \frac{-8MN}{A} = \frac{-8000kN}{0,4 \cdot 0,8} = 25000\ kN/m^2$

$E = 456\,Pa = 45000\ MPa = \underline{45 \cdot 10^6\ kN/m^2}$

$\nu = 0,2$

L → x
h → y
b → z

$\phi_v = \frac{1-2\nu}{6E}\sigma_x^2$ | $\frac{1-2\nu}{6E}\cdot\sigma_x^2 = 2,22 \cdot 10^{-9}\ \left(\frac{kN}{m^2}\right)^{-1}$

$\phi_f = \frac{1+\nu}{3E}\sigma_x^2$ | $\frac{1+\nu}{3E}\cdot\sigma_x^2 = 8,89 \cdot 10^{-9}\ \left(\frac{kN}{m^2}\right)^{-1}$

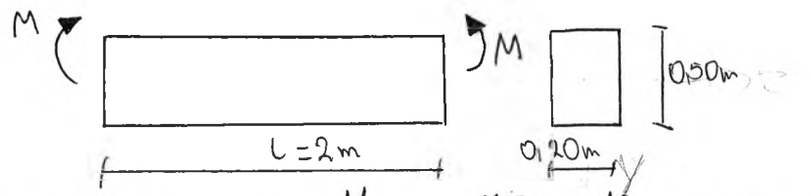
$$u_v = \int_V \phi_v\, dv = 2,22 \cdot 10^{-9} \int_0^{2,5}\int_{-0,4}^{0,4}\int_{-0,2}^{0,2} dx\,dy\,dz =$$

$$= 2,22 \cdot 10^{-9} \cdot x\Big|_0^{2,5} \cdot y\Big|_{-0,4}^{0,4} \cdot z\Big|_{-0,2}^{0,2} = 2,22 \cdot 10^{-9} \cdot 2,5 \cdot 0,8 \cdot 0,4 = \underline{1,11\ kNm}$$

$$u_f = \int_V \phi_f\, dv = 8,89 \cdot 10^{-9} \cdot 2,5 \cdot 0,8 \cdot 0,4 = \underline{4,45\ kNm}$$

$u = u_v + u_f$

$u = 5,56\ kNm$



$M = 7,5\ kNm \qquad \sigma = \frac{M}{J_z} \cdot y = \frac{7,5}{1} \cdot \frac{12}{0,2 \cdot 0,5^3} = 3600\ kN/m^3$

$E = 60\,GPa = 60 \cdot 10^6\ kN/m^2$

$\nu = 0,25$

$\phi_v = \frac{1-2\nu}{6E} \cdot 3600^2 \cdot y^2 = 0,018\,y^2$

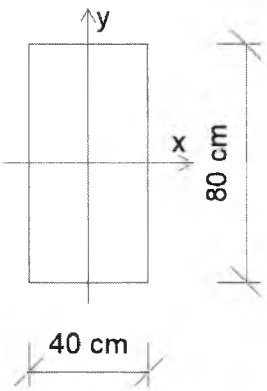
$\phi_f = \frac{1+\nu}{3E} \cdot 3600^2 \cdot y^2 = 0,09\,y^2$

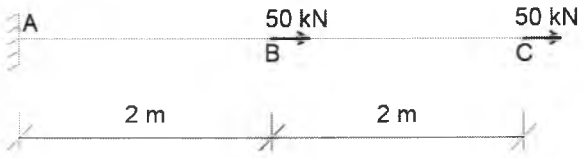
$$u_v = \int \phi_v\, dv = 0,018 \int_0^{2}\int_{-0,25}^{0,25}\int_{-0,1}^{0,1} y^2\, dx\,dy\,dz =$$

$$= 0,018 \cdot x\Big|_0^2 \cdot \frac{y^3}{3}\Big|_{-0,25}^{0,25} \cdot z\Big|_{-0,1}^{0,1} = 0,018 \cdot 2 \cdot \left(\frac{0,25^3}{3} + \frac{0,25^3}{3}\right) \cdot 0,1 = \underline{7,5 \times 10^{-5}\ [kNm]}$$

$$u_f = \int \phi_f\, dv = 0,09 \cdot 2 \cdot \left(\frac{0,25^3}{3} + \frac{0,25^3}{3}\right) \cdot 0,2 = \underline{3,75 \cdot 10^{-4}}$$

Oblicz całkowitą energię sprężystą układu





$$E = 205\ GPa$$

$$\nu = 0{,}2$$

$$\sigma_x^{BC} = \frac{50}{0{,}4 \cdot 0{,}8} = 156{,}25 \frac{kN}{m^2} = 0{,}156\ MPa$$

$$\sigma_x^{AB} = \frac{100}{0{,}4 \cdot 0{,}8} = 312{,}50 \frac{kN}{m^2} = 0{,}313\ MPa$$

$$\phi_v = \frac{1-2\nu}{6E}\left(\sigma_x + \sigma_y + \sigma_z\right)^2$$

$$\phi_v^{BC} = \frac{1-2\cdot 0{,}2}{6 \cdot 205 \cdot 10^3} \cdot 0{,}156^2 = 4{,}88 \cdot 10^{-7} \cdot 0{,}156^2 = 11{,}86 \cdot 10^{-9} \frac{1}{MPa}$$

$$\phi_v^{AB} = \frac{1-2\cdot 0{,}2}{6 \cdot 205 \cdot 10^3} \cdot 0{,}313^2 = 47{,}81 \cdot 10^{-9} \frac{1}{MPa}$$

$$U_v = \int_V \phi_v^{BC} dV + \int_V \phi_v^{AB} dV$$

$$= 11{,}86 \cdot 10^{-9} \int_0^{2000} \int_{-200}^{200} \int_{-400}^{400} dxdydz + 47{,}81 \cdot 10^{-9} \int_0^{2000} \int_{-200}^{200} \int_{-400}^{400} dxdydz$$

$$= (11{,}86 \cdot 10^{-9} + 47{,}81 \cdot 10^{-9}) \cdot 2000 \cdot 400 \cdot 800 = 38{,}19\ Nmm = 0{,}038\ J$$

$$\phi_f = \frac{1+\nu}{3E}\left[\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_x\sigma_y - \sigma_y\sigma_z - \sigma_x\sigma_z + 3\left(\tau_{xy}{}^2 + \tau_{yz}{}^2 + \tau_{xz}{}^2\right)\right]$$

$$\phi_f^{BC} = \frac{1+0{,}2}{3 \cdot 205 \cdot 10^3} \cdot 0{,}156^2 = 1{,}95 \cdot 10^{-6} \cdot 0{,}156^2 = 47{,}46 \cdot 10^{-9} \frac{1}{MPa}$$

$$\phi_f^{AB} = \frac{1+0{,}2}{3 \cdot 205 \cdot 10^3} \cdot 0{,}313^2 = 191{,}04 \cdot 10^{-9} \frac{1}{MPa}$$

$$U_f = (47{,}46 \cdot 10^{-9} + 191{,}04 \cdot 10^{-9}) \cdot 2000 \cdot 400 \cdot 800 = 152{,}64\ Nmm = 0{,}152\ J$$

$$U = U_v + U_f = 0{,}038 + 0{,}152 = 0{,}19\ J$$