Zad. 1. Dla zadanego pola przemieszczeń wyznacz odkształcenia główne w punkcie A=(0,8,10).

$$u = xy^2 \cdot 10^{-6}\, m$$

$$v = (2x + y^2 z) \cdot 10^{-6}\, m$$

$$\omega = (x + 2yz + z^2) \cdot 10^{-6}\, m$$

$$\varepsilon_x = \frac{\partial u}{\partial x} = y^2 \cdot 10^{-6} = 64 \cdot 10^{-6}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = 2yz \cdot 10^{-6} = 160 \cdot 10^{-6}$$

$$\varepsilon_z = \frac{\partial \omega}{\partial z} = (2y + 2z) \cdot 10^{-6} = 36 \cdot 10^{-6}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = (2xy + 2) \cdot 10^{-6} = 2 \cdot 10^{-6}$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial \omega}{\partial y} = (y^2 + 2z) \cdot 10^{-6} = 84 \cdot 10^{-6}$$

$$\gamma_{zx} = \frac{\partial \omega}{\partial x} + \frac{\partial u}{\partial z} = 1 \cdot 10^{-6}$$

$$T_\varepsilon^A = \begin{bmatrix} \varepsilon_x & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{yx}}{2} & \varepsilon_y & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{zx}}{2} & \frac{\gamma_{zy}}{2} & \varepsilon_z \end{bmatrix} = \begin{bmatrix} 64 & 1 & 0{,}5 \\ 1 & 160 & 42 \\ 0{,}5 & 42 & 36 \end{bmatrix} \cdot 10^{-6}$$

$$\varepsilon^3 - I_1 \varepsilon^2 + I_2 \varepsilon - I_3 = 0$$

$$I_1 = \varepsilon_x + \varepsilon_y + \varepsilon_z = (64 + 160 + 36) \cdot 10^{-6} = 260 \cdot 10^{-6}$$

$$I_2 = \begin{vmatrix} 160 & 42 \\ 42 & 36 \end{vmatrix} + \begin{vmatrix} 64 & 0{,}5 \\ 0{,}5 & 36 \end{vmatrix} + \begin{vmatrix} 64 & 1 \\ 1 & 160 \end{vmatrix} = 16538 \cdot 10^{-12}$$

$$I_3 = \begin{vmatrix} 64 & 1 & 0{,}5 \\ 1 & 160 & 42 \\ 0{,}5 & 42 & 36 \end{vmatrix} = 255710 \cdot 10^{-18}$$

$$\varepsilon^3 - 260 \cdot 10^{-6} \cdot \varepsilon^2 + 16538 \cdot 10^{-12} \cdot \varepsilon - 255710 \cdot 10^{-18} = 0$$

$$\varepsilon_1 = 172{,}91 \cdot 10^{-6};\ \varepsilon_2 = 63{,}98 \cdot 10^{-6};\ \varepsilon_3 = 23{,}12 \cdot 10^{-6}$$

$$T_{\varepsilon 0}^A = \begin{bmatrix} 172{,}91 & 0 & 0 \\ 0 & 63{,}98 & 0 \\ 0 & 0 & 23{,}12 \end{bmatrix} \cdot 10^{-6}$$

1. Oblicz i zapisz macierz odkształceń w punkcie A(2 , 1 , 8).

Dla zadanego pola przemieszczeń:

$u = 2xy + 6z$

$v = 4x + 2yz$

$w = 5xyz$

$$T_\varepsilon = \begin{vmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_z \end{vmatrix}$$

$$\varepsilon_x = \frac{\delta u}{\delta x} = 2y = 2$$

$$\varepsilon_y = \frac{\delta v}{\delta y} = 2z = 16$$

$$\varepsilon_z = \frac{\delta w}{\delta z} = 5xy = 10$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\delta u}{\delta y} + \frac{\delta v}{\delta x}\right) = \frac{1}{2}(2x+4) = \frac{1}{2}(4+4) = 4$$

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\delta u}{\delta z} + \frac{\delta w}{\delta x}\right) = \frac{1}{2}(6+5yz) = \frac{1}{2}(6+40) = 23$$

$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\delta w}{\delta y} + \frac{\delta v}{\delta z}\right) = \frac{1}{2}(2y+5xz) = \frac{1}{2}(2+80) = 41$$

$$T_\varepsilon{}^A = \begin{vmatrix} 2 & 4 & 23 \\ 4 & 16 & 41 \\ 23 & 41 & 10 \end{vmatrix}$$

2. Obliczyć odkształcenia główne.

$$\boldsymbol{\varepsilon^3 - I_1\varepsilon^2 + I_2\varepsilon - I_3 = 0}$$

$$I_1 = \varepsilon_x + \varepsilon_y + \varepsilon_z = 2 + 16 + 10 = \mathbf{28}$$

$$I_2 = \begin{vmatrix} \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{zy} & \varepsilon_z \end{vmatrix} + \begin{vmatrix} \varepsilon_x & \varepsilon_{xz} \\ \varepsilon_{zx} & \varepsilon_z \end{vmatrix} + \begin{vmatrix} \varepsilon_x & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_y \end{vmatrix} = \begin{vmatrix} 16 & 41 \\ 41 & 10 \end{vmatrix} + \begin{vmatrix} 2 & 23 \\ 23 & 10 \end{vmatrix} + \begin{vmatrix} 2 & 4 \\ 4 & 16 \end{vmatrix} =$$

$$= 16\cdot 10 - 41\cdot 41 + 2\cdot 10 - 23\cdot 23 + 2\cdot 16 - 4\cdot 4 = \mathbf{-2014}$$

$$I_3 = \begin{vmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_z \end{vmatrix} = \begin{vmatrix} 2 & 4 & 23 \\ 4 & 16 & 41 \\ 2 & 41 & 10 \end{vmatrix} =$$

$$= 2\cdot 16\cdot 10 + 4\cdot 41\cdot 23 + 23\cdot 4\cdot 41 - 23\cdot 16\cdot 23 - 2\cdot 41\cdot 41 - 4\cdot 4\cdot 10 = \mathbf{-4122}$$

$$\varepsilon^3 - 28\cdot\varepsilon^2 - 2014\cdot\varepsilon + 4122 = 0$$

$\varepsilon_1 = 60{,}278$

$\varepsilon_2 = 1{,}995$

$\varepsilon_3 = -34{,}273$

$$T_\varepsilon{}^0 = \begin{vmatrix} 60{,}278 & 0 & 0 \\ 0 & 1{,}995 & 0 \\ 0 & 0 & -34{,}273 \end{vmatrix}$$

$$I_1{}^0 = 60{,}278 + 1{,}995 - 34{,}273 = \mathbf{28} \approx I_1$$

$$I_2{}^0 = 60{,}278\cdot 1{,}995 - 1{,}995\cdot 34{,}273 - 60{,}278\cdot 34{,}273 = \mathbf{-2014{,}03} \approx I_2$$

$$I_3{}^0 = 60{,}278\cdot 1{,}995\cdot(-34{,}273) = \mathbf{-4121{,}49} \approx I_3$$

3. Oblicz i zapisz macierz naprężeń w punkcie A.

$E = 2{,}1\cdot 10^5 MPa$

$\nu = 0{,}25$

$$T_\varepsilon{}^A = \begin{vmatrix} 2 & 4 & 23 \\ 4 & 16 & 41 \\ 23 & 41 & 10 \end{vmatrix}$$

$$T_\sigma{}^A = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{vmatrix}$$

$$G = \frac{E}{2(1+\nu)} = \frac{2{,}1\cdot 10^5}{2(1+0{,}25)} = 84000\,MPa$$

$$\lambda = \frac{E\cdot\nu}{(1+\nu)(1-2\nu)} = \frac{2{,}1\cdot 10^5\cdot 0{,}25}{(1+0{,}25)(1-2\cdot 0{,}25)} = 84000\,MPa$$

$$\sigma_x = 2G\varepsilon_x + \lambda(\varepsilon_x + \varepsilon_y + \varepsilon_z) = 2\cdot 84000\cdot 2 + 84000\cdot 28 = 0{,}269\cdot 10^7\,MPa$$

$$\sigma_y = 2G\varepsilon_y + \lambda(\varepsilon_x + \varepsilon_y + \varepsilon_z) = 2\cdot 84000\cdot 16 + 84000\cdot 28 = 0{,}504\cdot 10^7\,MPa$$

$$\sigma_z = 2G\varepsilon_z + \lambda(\varepsilon_x + \varepsilon_y + \varepsilon_z) = 2\cdot 84000\cdot 10 + 84000\cdot 28 = 0{,}403\cdot 10^7\,MPa$$

$$\tau_{xy} = 2G\varepsilon_{xy} = 2\cdot 84000\cdot 4 = 0{,}0672\cdot 10^7\ \mathrm{MPa}$$

$$\tau_{xz} = 2G\varepsilon_{xz} = 2\cdot 84000\cdot 23 = 0{,}3864\cdot 10^7\mathrm{MPa}$$

$$\tau_{yz} = 2G\varepsilon_{yz} = 2\cdot 84000\cdot 41 = 0{,}6888\cdot 10^7\ \mathrm{MPa}$$

$$T_\sigma{}^A = \begin{vmatrix} 0{,}269 & 0{,}0672 & 0{,}3864 \\ 0{,}0672 & 0{,}504 & 0{,}6888 \\ 0{,}3864 & 0{,}6888 & 0{,}403 \end{vmatrix}\cdot 10^7\ \mathrm{MPa}$$

Dla zadanego pola przemieszczeń

$u = 12 + 6xy + 8z$

$v = 4 + 12x + 6yz$

$w = 8 + 4xyz$

1. **Oblicz i zapisz macierz odkształceń w punkcie A(2, 1, 2)**

Odkształcenia linowe:

$$\varepsilon_x = \frac{\partial u}{\partial x} = (12 + 6xy + 8z)' = 6y = 6 \cdot 1 = 6$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = (4 + 12x + 6yz)' = 6z = 6 \cdot 2 = 12$$

$$\varepsilon_z = \frac{\partial w}{\partial z} = (8 + 4xyz)' = 4xy = 4 \cdot 2 \cdot 1 = 8$$

Odkształcenia kątowe:

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial_u}{\partial_y} + \frac{\partial_v}{\partial_x}\right) = \frac{1}{2}(6x + 12) = \frac{1}{2}(6 \cdot 2 + 12) = 12$$

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial_u}{\partial_z} + \frac{\partial_w}{\partial_x}\right) = \frac{1}{2}(8 + 4yz) = \frac{1}{2}(8 + 4 \cdot 1 \cdot 2) = 8$$

$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial_v}{\partial_z} + \frac{\partial_w}{\partial_y}\right) = \frac{1}{2}(6y + 4xz) = \frac{1}{2}(6 \cdot 1 + 4 \cdot 2 \cdot 2) = 11$$

Tensor odkształceń:

$$T_\varepsilon^A = \begin{bmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{yz} & \varepsilon_z \end{bmatrix} = \begin{bmatrix} 6 & 12 & 8 \\ 12 & 12 & 11 \\ 8 & 11 & 8 \end{bmatrix}$$ ✓

2. **Wyznaczenie odkształceń głównych**

Niezmienniki

$$J_1^\varepsilon = \varepsilon_x + \varepsilon_y + \varepsilon_z = 6 + 12 + 8 = 26$$

$$J_2^\varepsilon = \begin{bmatrix} \varepsilon_x & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_y \end{bmatrix} + \begin{bmatrix} \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{zy} & \varepsilon_z \end{bmatrix} + \begin{bmatrix} \varepsilon_x & \varepsilon_{xz} \\ \varepsilon_{zx} & \varepsilon_z \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 12 & 12 \end{bmatrix} + \begin{bmatrix} 12 & 11 \\ 11 & 8 \end{bmatrix} + \begin{bmatrix} 6 & 8 \\ 8 & 8 \end{bmatrix}$$

$$J_2^\varepsilon = 6 \cdot 12 - 12 \cdot 12 + 12 \cdot 8 - 11 \cdot 11 + 6 \cdot 8 - 8 \cdot 8 = -113$$

$$J_3^\varepsilon = det \begin{bmatrix} 6 & 12 & 8 \\ 12 & 12 & 11 \\ 8 & 11 & 8 \end{bmatrix} \begin{matrix} 6 & 12 \\ 12 & 12 \\ 8 & 11 \end{matrix}$$
$$= 6 \cdot 12 \cdot 8 + 12 \cdot 11 \cdot 8 + 8 \cdot 12 \cdot 11 - 8 \cdot 12 \cdot 8 - 11 \cdot 11 \cdot 6 - 8 \cdot 12 \cdot 12 = 42$$

Równanie wiekowe:

$$\varepsilon^3 - J_1^\varepsilon \cdot \varepsilon^2 + J_2^\varepsilon \cdot \varepsilon - J_3^\varepsilon = 0$$

$$\varepsilon^3 - 26 \cdot \varepsilon^2 - 113 \cdot \varepsilon - 42 = 0$$

3. **Oblicz i zapisz macierz naprężeń w punkcie „A"**

$E = 2{,}1 \cdot 10^5\ [MPa]$

$\nu = 0{,}25$

$$G = \frac{E}{2(1+\nu)} = \frac{2{,}1 \cdot 10^5}{2(1 + 0{,}25)} = 8{,}4 \cdot 10^4 [MPa]$$

$$\lambda = \frac{E \cdot \nu}{(1+\nu)\cdot(1-2\nu)} = \frac{2{,}1 \cdot 10^5 \cdot 0{,}25}{(1 + 0{,}25)\cdot(1 - 2 \cdot 0{,}25)} = 8{,}4 \cdot 10^4 [MPa]$$

$$\sigma_x = 2 \cdot G \cdot \varepsilon_x + \lambda(\varepsilon_x + \varepsilon_y + \varepsilon_z)$$

$$\sigma_x = 2 \cdot 8{,}4 \cdot 10^4 \cdot 6 + 8{,}4 \cdot 10^4 (6 + 12 + 8) = 23{,}52 \cdot 10^5 [MPa]$$

$$\sigma_y = 2 \cdot G \cdot \varepsilon_y + \lambda(\varepsilon_x + \varepsilon_y + \varepsilon_z)$$

$$\sigma_y = 2 \cdot 8{,}4 \cdot 10^4 \cdot 12 + 8{,}4 \cdot 10^4 (6 + 12 + 8) = 42{,}0 \cdot 10^5 [MPa]$$

$$\sigma_z = 2 \cdot G \cdot \varepsilon_z + \lambda(\varepsilon_x + \varepsilon_y + \varepsilon_z)$$

$$\sigma_z = 2 \cdot 8{,}4 \cdot 10^4 \cdot 8 + 8{,}4 \cdot 10^4 (6 + 12 + 8) = 35{,}28 \cdot 10^5 [MPa]$$

$$\tau_{xy} = 2 \cdot G \cdot \varepsilon_{xy} = 2 \cdot 8{,}4 \cdot 10^4 \cdot 12 = 20{,}16 \cdot 10^5 [MPa]$$

$$\tau_{xz} = 2 \cdot G \cdot \varepsilon_{xz} = 2 \cdot 8{,}4 \cdot 10^4 \cdot 8 = 13{,}44 \cdot 10^5 [MPa]$$

$$\tau_{yz} = 2 \cdot G \cdot \varepsilon_{yz} = 2 \cdot 8{,}4 \cdot 10^4 \cdot 11 = 18{,}48 \cdot 10^5 [MPa]$$

$$T_\sigma^A = \begin{bmatrix} \sigma_1 & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_2 & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_3 \end{bmatrix} = \begin{bmatrix} 23{,}52 & 20{,}16 & 13{,}44 \\ 20{,}16 & 42{,}0 & 18{,}48 \\ 13{,}44 & 18{,}48 & 35{,}28 \end{bmatrix} \cdot 10^5 [MPa]$$

$\varepsilon_1 > \varepsilon_2 > \varepsilon_3$

$$\varepsilon = \begin{bmatrix} 29{,}83 & 0 & 0 \\ 0 & -3{,}42 & 0 \\ 0 & 0 & -0{,}41 \end{bmatrix}$$

$$I_1 = 29{,}83 - 3{,}42 - 0{,}41 = 26$$

$$I_2 == \begin{bmatrix} -3{,}42 & 0 \\ 0 & -0{,}41 \end{bmatrix} + \begin{bmatrix} 29{,}83 & 0 \\ 0 & -0{,}41 \end{bmatrix} + \begin{bmatrix} 29{,}83 & 0 \\ 0 & -3{,}42 \end{bmatrix} = 1{,}40 - 12{,}23 - 102{,}02 = 112{,}85$$

$$I_3 = \begin{bmatrix} 29{,}83 & 0 & 0 \\ 0 & -3{,}42 & 0 \\ 0 & 0 & -0{,}41 \end{bmatrix} = -41{,}83$$

Dane:

$u = (3xy + 2z) * 10^{-6}$

$v = (8x + yz) * 10^{-6}$

$w = (5xyz) * 10^{-6}$

A(2,4,2)

1/. Macierz odkształceń w punkcie A(2,4,2)

$$T_\varepsilon^A = \begin{bmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_z \end{bmatrix}$$

$$\varepsilon_x = \frac{\delta_u}{\delta_x} = 3y * 10^{-6} = 12 * 10^{-6}$$

$$\varepsilon_y = \frac{\delta_v}{\delta_y} = z * 10^{-6} = 2 * 10^{-6}$$

$$\varepsilon_z = \frac{\delta_w}{\delta_z} = 5xy * 10^{-6} = 40 * 10^{-6}$$

$$\gamma_{xy} = \frac{\delta_u}{\delta_y} + \frac{\delta_v}{\delta_x} = (3x + 8) * 10^{-6} = 14 * 10^{-6}$$

$$\gamma_{xz} = \frac{\delta_u}{\delta_z} + \frac{\delta_w}{\delta_x} = (2 + 5yz) * 10^{-6} = 42 * 10^{-6}$$

$$\gamma_{yz} = \frac{\delta_v}{\delta_z} + \frac{\delta_w}{\delta_y} = (y + 5xz) * 10^{-6} = 24 * 10^{-6}$$

$$T_\varepsilon^A = \begin{bmatrix} 12 & 7 & 21 \\ 7 & 2 & 12 \\ 21 & 12 & 40 \end{bmatrix} * 10^{-6}$$

2/. Odkształcenia główne:

$$\varepsilon^3 - I_1\varepsilon^2 + I_2\varepsilon - I_3 = 0$$

$I_1 = 12 + 2 + 40 = 54 * 10^{-6}$

$$I_2 = \begin{vmatrix} 2 & 12 \\ 12 & 40 \end{vmatrix} + \begin{vmatrix} 12 & 21 \\ 21 & 40 \end{vmatrix} + \begin{vmatrix} 12 & 7 \\ 7 & 2 \end{vmatrix} = 80 - 144 + 480 - 441 + 24 - 49 = -50 * 10^{-12}$$

$$I_3 = \begin{vmatrix} 12 & 7 & 21 \\ 7 & 2 & 12 \\ 21 & 12 & 40 \end{vmatrix} = 12 * 2 * 40 + 7 * 12 * 21 + 21 * 7 * 12 - $$

$$-21 * 2 * 21 - 12 * 12 * 12 - 40 * 7 * 7 = -82 * 10^{-18}$$

$$\varepsilon^3 - 54 * 10^{-6}\varepsilon^2 - 50 * 10^{-12}\varepsilon + 82 * 10^{-18} = 0$$

$\varepsilon_1 = 54{,}88 * 10^{-6}$

$\varepsilon_2 = 0{,}86 * 10^{-6}$

$\varepsilon_3 = -1{,}74 * 10^{-6}$

$$T_\varepsilon^{A0} = \begin{bmatrix} 54{,}88 & 0 & 0 \\ 0 & 0{,}86 & 0 \\ 0 & 0 & -1{,}74 \end{bmatrix} * 10^{-6}$$

Sprawdzenie:

$I_1^0 = (54{,}88 + 0{,}86 - 1{,}74) * 10^{-6} = 54 * 10^{-6}$

$$I_2^0 = \begin{vmatrix} 0{,}86 & 0 \\ 0 & -1{,}74 \end{vmatrix} * 10^{-6} + \begin{vmatrix} 54{,}88 & 0 \\ 0 & -1{,}74 \end{vmatrix} * 10^{-6} + \begin{vmatrix} 54{,}88 & 0 \\ 0 & 0{,}86 \end{vmatrix} * 10^{-6} =$$

$$= (-1{,}50 - 95{,}49 + 47{,}20) * 10^{-12} = -49{,}79 * 10^{-12}$$

$$I_3^0 = \begin{vmatrix} 54{,}88 & 0 & 0 \\ 0 & 0{,}86 & 0 \\ 0 & 0 & -1{,}74 \end{vmatrix} * 10^{-6} = (-82{,}12 + 0) * 10^{-18} = -82{,}12 * 10^{-18}$$

3/. Macierz naprężeń w punkcie A

$$T_\sigma^A = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

$$\sigma_x = \frac{E}{1+v}\left[\varepsilon_x + \frac{v}{1-2v} * (\varepsilon_x + \varepsilon_y + \varepsilon_z)\right] =$$

$$= \frac{2{,}1 * 10^5}{1 + 0{,}25}\left[12 * 10^{-6} + \frac{0{,}25}{1 - 2 * 0{,}25} * (12 + 2 + 40) * 10^{-6}\right] = 6{,}55\ [MPa]$$

$$\sigma_y = \frac{E}{1+v}\left[\varepsilon_y + \frac{v}{1-2v} * (\varepsilon_x + \varepsilon_y + \varepsilon_z)\right] =$$

$$= \frac{2{,}1 * 10^5}{1 + 0{,}25}\left[2 * 10^{-6} + \frac{0{,}25}{1 - 2 * 0{,}25} * (12 + 2 + 40) * 10^{-6}\right] = 4{,}87\ [MPa]$$

$$\sigma_z = \frac{E}{1+v}\left[\varepsilon_z + \frac{v}{1-2v} * (\varepsilon_x + \varepsilon_y + \varepsilon_z)\right] =$$

$$= \frac{2{,}1 * 10^5}{1 + 0{,}25}\left[40 * 10^{-6} + \frac{0{,}25}{1 - 2 * 0{,}25} * (12 + 2 + 40) * 10^{-6}\right] = 11{,}26\ [MPa]$$

$$G = \frac{E}{2 * (1 + v)} = \frac{2{,}1 * 10^5}{2 * (1 + 0{,}25)} = 84 * 10^3$$

$$\tau_{xy} = G * \gamma_{xy} = 84 * 10^3 * 14 * 10^{-6} = 1{,}18\ [MPa]$$

$$\tau_{xz} = G * \gamma_{xz} = 84 * 10^3 * 42 * 10^{-6} = 3{,}53\ [MPa]$$

$$\tau_{yz} = G * \gamma_{yz} = 84 * 10^3 * 24 * 10^{-6} = 2{,}02\ [MPa]$$

$$T_\sigma^A = \begin{bmatrix} 6{,}55 & 1{,}18 & 3{,}53 \\ 1{,}18 & 4{,}87 & 2{,}02 \\ 3{,}53 & 2{,}02 & 11{,}26 \end{bmatrix} MPa$$